Jacobi's method

1. Use four steps of Jacobi's method to approximate a solution to a system of linear equations $A\mathbf{u} = \mathbf{v}$ where

$$A = \begin{pmatrix} 5 & 1 & -2 \\ 1 & 10 & 2 \\ -2 & 2 & 10 \end{pmatrix} \text{ and } \mathbf{v} = \begin{pmatrix} -1.5 \\ 0.2 \\ -1.0 \end{pmatrix}.$$

Answer: $\mathbf{u}_0 = A_{\text{diag}}^{-1}\mathbf{v} = \begin{pmatrix} -0.3 \\ 0.02 \\ -0.1 \end{pmatrix}$, and $\mathbf{u}_1 = \begin{pmatrix} -0.344 \\ 0.07 \\ -0.164 \end{pmatrix}$, $\mathbf{u}_2 = \begin{pmatrix} -0.3796 \\ 0.0872 \\ -0.1828 \end{pmatrix}$, $\mathbf{u}_3 = \begin{pmatrix} -0.39056 \\ 0.09452 \\ -0.19336 \end{pmatrix}$, $\mathbf{u}_4 = \begin{pmatrix} -0.396248 \\ 0.097728 \\ -0.197016 \end{pmatrix}$.

2. The solution to Question 1 is the vector $\mathbf{u} = \begin{pmatrix} -0.4 \\ 0.1 \\ -0.2 \end{pmatrix}$. What is $\|\mathbf{u} - \mathbf{u}_k\|_2$ for each of these approximations? Here, $\|\mathbf{v}\|_2$ represents the 2-norm of the vector $\mathbf{v}$, also known as the *Euclidean* norm: the square root of the sum of the squares of the absolute values of the entries.

Answer: 0.1625, 0.07302, 0.02959, 0.01278, 0.005305

3. The errors in each approximation in Question 2 seem to drop by approximately a constant with each step. What would be your estimate as to the reduction in this error?

Answer: The appears to drop by a value between 2.0 and 2.5, but 2.37 is close.

4. Verify your response to Question 3 by running the following Matlab code:

```
A = [5 1 -2; 1 10 2; -2 2 10];
v = [-1.5 0.2 -1.0]';
u = [-0.4 0.1 -0.2]';           # The exact solution to A*u = v
Adiag = diag(diag(A));
Aoff  = A - Adiag;
InvAdiag = Adiag^-1;
u1 = InvAdiag*v;

for i = 1:50
    u0 = u1;
    u1 = InvAdiag*(v - Aoff*u1);
    norm( u0 - u )/norm( u1 - u )
end
```

5. What is happening in the last few steps of the for loop in Question 4?

Acknowledgement: Chinemerem Chigbo pointed out they may not teach the 2-norm representation in first-year linear algebra.